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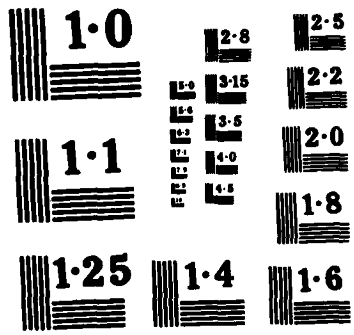
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ON ASYMPTOTIC DISTRIBUTION OF THE TEST STATISTIC FOR THE MEAN OF THE NON-ISOTROPIC PRINCIPAL COMPONENT

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ABSTRACT

In this paper, the authors derived the large sample distribution of the t statistic based upon the observations on the first principal component instead of the original variables. It is shown that the above statistic is distributed asymptotically as Student's t distribution.

Key Words and Phrases: Principal components and asymptotic distribution.

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1. INTRODUCTION

Data analysts are often confronted with the problem of large dimensional data. In some of these situations, it is customary to reduce the dimensionality of the problem by using principal component analysis and to perform statistical analysis of the data using the new variables (principal components). For example, the new variables are used in the area of classification. Chestnut and Floyd (1981) used the principal components as variables in identification of underwater targets. However, the statistical data analysis using the principal components is adhoc since the distributions of the test statistics based upon the principal components are complicated when the covariance matrix is unknown. Very little work was done in the literature on deriving the distributions of these test statistics even in the asymptotic case. In this paper, we derive the asymptotic distribution of the t statistic based upon the new variable (the most important principal component) instead of using any of the original variables. The above asymptotic distribution is shown to be Student's t distribution. The accuracy of the above approximation is studied by comparing the simulated values using the asymptotic expression with the standard Student's t table. It is found that the accuracy of the above approximation is sufficient for many practical situations.

2. ASYMPTOTIC DISTRIBUTION OF t-STATISTIC BASED UPON A PRINCIPAL COMPONENT

Consider a random matrix $X = (\underset{\sim}{X}_1,\ldots,\underset{\sim}{X}_{n+1}): p\times(n+1)$ whose columns are distributed independently as multivariate normal with a common covariance matrix Σ and mean vector $\underset{\sim}{\mu}$. Now,

$$E(S/n) = \Sigma \tag{2.1}$$

where $S = \sum_{i=1}^{n+1}(\underset{\sim}{X}_i-\bar{\underset{\sim}{X}})(\underset{\sim}{X}_i-\bar{\underset{\sim}{X}})'$, $\bar{\underset{\sim}{X}} = \sum_{i=1}^{n+1}\underset{\sim}{X}_i/(n+1)$. Let Γ: $p\times p$ be an orthogonal matrix such that $\Gamma'\Sigma\,\Gamma = \Lambda = \mathrm{diag}(\lambda_1,\ldots,\lambda_p)$ and $\lambda_1 \geq \ldots \geq \lambda_p$. Also, let G be an orthogonal matrix such that $\frac{G'SG}{n} = L = \mathrm{diag}(\ell_1,\ldots,\ell_p)$ and $\ell_1 \geq \ldots \geq \ell_p$. Now, let

$$Y = \sqrt{n}((S/n) - \Sigma) \tag{2.2}$$

so that

$$\frac{\Gamma' S\Gamma}{n} = \Lambda + Z \tag{2.3}$$

where $Z = \frac{\Gamma' Y\Gamma}{\sqrt{n}} = (z_{ij})$. So,

$$\Lambda H + ZH = HL \tag{2.4}$$

where $H = \Gamma' G$. Now, let $\Gamma = (\gamma_{ij})$ and $G = (g_{ij})$. It is known (see Mallows (1961), Fang and Krishnaiah (1981)) by applying perturbation technique that for $\lambda_{\alpha-1} > \lambda_\alpha > \lambda_{\alpha+1}$,

$$\ell_\alpha = \lambda_\alpha + z_{\alpha\alpha} + \sum_{i\neq\alpha}\frac{z_{\alpha i}^2}{\lambda_\alpha-\lambda_i} + 0(n^{-3/2})$$

$$h_{j\alpha} = \frac{z_{j\alpha}}{\lambda_\alpha-\lambda_j} + \sum_{m\neq\alpha}\frac{z_{jm}z_{m\alpha}}{(\lambda_\alpha-\lambda_m)(\lambda_\alpha-\lambda_j)} - \frac{z_{j\alpha}z_{\alpha\alpha}}{(\lambda_\alpha-\lambda_j)^2} + 0(n^{-3/2}),\quad j\neq\alpha$$

$$h_{\alpha\alpha} = 1 - \frac{1}{2}\sum_{m\neq\alpha}\frac{z_{\alpha m}z_{m\alpha}}{(\lambda_\alpha-\lambda_m)^2} + 0(n^{-3/2}) \tag{2.5}$$

where

$$Z_{ij} = \frac{1}{\sqrt{n}} a_{ij} = \frac{1}{\sqrt{n}} \sum_{\ell,k}^{p} \gamma_{\ell i}\gamma_{kj}Y_{\ell k}. \tag{2.6}$$

Using $H = \Gamma' G$, we obtain

$$\begin{aligned} g_{j\alpha} &= \sum_{m=1}^{p} \gamma_{jm} h_{m\alpha} \\ &= \gamma_{j\alpha} + \frac{1}{\sqrt{n}} \sum_{m\neq\alpha} \gamma_{jm} \frac{a_{m\alpha}}{\lambda_\alpha - \lambda_m} \\ &+ \frac{1}{n}\Big[\sum_{m\neq\alpha} \sum_{i\neq\alpha} \gamma_{jm} \frac{a_{mi}a_{i\alpha}}{(\lambda_\alpha-\lambda_i)(\lambda_\alpha-\lambda_m)} - \sum_{m\neq\alpha} \gamma_{jm} \frac{a_{m\alpha}a_{\alpha\alpha}}{(\lambda_\alpha-\lambda_m)^2} \\ &- \frac{1}{2} \sum_{m\neq\alpha} \gamma_{j\alpha} \frac{a_{\alpha m}^2}{(\lambda_\alpha-\lambda_m)^2}\Big] + O(n^{-3/2}) \\ &= \gamma_{j\alpha} + g_{j\alpha}(n^{-1/2}) + g_{j\alpha}(n^{-1}) + O(n^{-3/2}). \end{aligned} \tag{2.7}$$

Under the assumption of a single non-isotropic principal component, the eigenvalue λ_1 is simple. Let the corresponding eigenvector be denoted by $\underset{\sim}{\Gamma}_1$. Let $\underset{\sim}{g}_1 = (g_{11},\ldots,g_{p1})'$ be the sample eigenvector corresponding to the largest eigenvalue ℓ_1 of S/n, and

$$\underset{\sim}{g}_1 = \underset{\sim}{\Gamma}_1 + \underset{\sim}{g}_1(n^{-1/2}) + \underset{\sim}{g}_1(n^{-1}) + O(n^{-3/2}) \tag{2.8}$$

according to Eq. (2.7). Now consider the statistic

$$T = \sqrt{n}\ \underset{\sim}{g}_1'(\underset{\sim}{\bar{X}} - \underset{\sim}{\mu})/\sqrt{\underset{\sim}{g}_1' S \underset{\sim}{g}_1/n}\,. \tag{2.9}$$

We know that

$$\begin{aligned} \sqrt{n}\ \underset{\sim}{g}_1'(\underset{\sim}{\bar{X}} - \underset{\sim}{\mu}) &= \sqrt{n}\ \underset{\sim}{\Gamma}_1'(\underset{\sim}{\bar{X}} - \underset{\sim}{\mu}) + \underset{\sim}{g}_1'(n^{-1/2})\sqrt{n}(\underset{\sim}{\bar{X}} - \underset{\sim}{\mu}) + \ldots \\ &= \sqrt{n}\ \underset{\sim}{\Gamma}_1'(\underset{\sim}{\bar{X}} - \underset{\sim}{\mu}) + o_p(1) \end{aligned} \tag{2.10}$$

$$(\underset{\sim}{g}_1' S \underset{\sim}{g}_1/n)^{-1/2} = (\underset{\sim}{\Gamma}_1' S \underset{\sim}{\Gamma}_1/n)^{-1/2}$$

$$\times \left[1 - \frac{1}{2}\left(\frac{2\underset{\sim}{\Gamma}_1' S \underset{\sim}{g}_1(n^{-1/2})}{\underset{\sim}{\Gamma}_1' S \underset{\sim}{\Gamma}_1} + \frac{2\Gamma_1' S \underset{\sim}{g}_1(n^{-1})}{\underset{\sim}{\Gamma}_1' S \underset{\sim}{\Gamma}_1}\right.\right.$$

$$\left.\left. + \frac{g_1'(n^{-1/2}) S g_1(n^{-1/2})}{\underset{\sim}{\Gamma}_1' S \underset{\sim}{\Gamma}_1}\right) + \ldots\right]$$

$$= (\underset{\sim}{\Gamma}_1' S \underset{\sim}{\Gamma}_1/n)^{-1/2} + o_p(1). \qquad (2.11)$$

Since $\sqrt{n}\,(\bar{\underset{\sim}{X}} - \underset{\sim}{\mu})$ is of order $O_p(1)$, and the Y_{ij}'s in $\underset{\sim}{g}_1'(n^{-r/2})$, $r = 1,2,\ldots,$ are also of order $O_p(1)$, the order of probability convergence in Eq. (2.10), (2.11) is valid according to the Chernoff-Pratt definition of o_p (Bishop, Fienberg and Holland (1975)).

The statistic

$$T = \frac{\sqrt{n}\underset{\sim}{g}_1'(\bar{\underset{\sim}{X}} - \underset{\sim}{\mu})}{\sqrt{\underset{\sim}{g}_1' S \underset{\sim}{g}_1/n}} = \frac{\sqrt{n}\ \underset{\sim}{\Gamma}_1'(\bar{\underset{\sim}{X}} - \underset{\sim}{\mu})}{\sqrt{\underset{\sim}{\Gamma}_1' S \underset{\sim}{\Gamma}_1/n}} + o_p(1). \qquad (2.12)$$

So the statistic T converges in distribution to Student's t distribution with n degrees-of-freedom.

Suppose, we wish to test the hypothesis that $\underset{\sim}{\Gamma}_1'\underset{\sim}{\mu} = 0$. Then, we use

$$T = \frac{\sqrt{n}\ \underset{\sim}{g}_1'\bar{\underset{\sim}{X}}}{\sqrt{\underset{\sim}{g}_1' S \underset{\sim}{g}_1/n}}$$

as a test statistic.

3. AN EMPIRICAL STUDY ON THE ACCURACY OF THE APPROXIMATION

In this section, we study the accuracy of the asymptotic expression given in the preceding section. In Table 1, the entries in the rows corresponding to t_α give the values of t_α where

$$P[t \leq t_\alpha] = (1-\alpha) \tag{3.1}$$

and t is distributed as Student's t distribution with n degrees of freedom. The entries in the rows corresponding to $\hat{\alpha}$ are the simulated values of α obtained by using the IMSL subroutines GGNSM, EIGRS for the Monte Carlo methods. In computing the simulated values, 5000 trials are performed and each trial consisted of a random sample of size n+1 from a multivariate normal population with covariance matrix $\Sigma = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$. The entries in the table are computed for different values of n, λ_1, λ_2, λ_3 and p. From the table, we observe that the approximation is satisfactory when n is moderately large like 23. The approximation is not good when α is small and n = 10. But, the accuracy of the approximation increased as α increased even when n = 10. From Tables 2 and 3, we observe that the approximation is good when n = 23 and α increases for p = 4,5.

TABLE 1

COMPARISON OF ASYMPTOTIC SIGNIFICANCE LEVELS OF t WITH SIMULATED VALUES WHEN p = 3

$(\lambda_1,\lambda_2,\lambda_3) = (3,1,1)$ n = 10

α	.005	.01	.025	.05	.1	.15	.2	.25	.3	.35	.4	.45	.5
t_α	-3.169	-2.764	-2.228	-1.812	-1.372	-1.093	-.879	-.700	-.542	-.397	-.260	-.129	0.0
Simu. $\hat{\alpha}$	.0012	.0046	.0128	.0322	.0722	.1206	.1722	.2272	.2862	.3352	.3922	.4500	.5104
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0010	.0019	.0032	.0050	.0073	.0092	.0107	.0119	.0128	.0134	.0138	.0141	.0141
α	.55	.6	.65	.7	.75	.8	.85	.9	.95	.975	.99	.995	
t_α	.129	.260	.397	.542	.700	.879	1.093	1.372	1.812	2.228	2.764	3.169	
Simu. $\hat{\alpha}$	.5670	.6224	.6770	.7338	.7884	.8368	.8854	,9288	.97	.9888	.9974	.9982	
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0140	.0137	.0132	.0125	.0116	.0105	.009	.0073	.0048	.0030	.0014	.0012	

TABLE 1 (continued)

$(\lambda_1,\lambda_2,\lambda_3) = (3,1,1)$ n = 23

α	.005	.01	.025	.05	.1	.15	.2	.25	.3	.35	.4	.45	.5
t_α	-2.807	-2.500	-2.069	-1.714	-1.319	-1.060	-.858	-.685	-.532	-.390	-.256	-.127	0.0
Simu. $\hat{\alpha}$	.0034	.007	.0210	.0426	.0918	.144	.1930	.2440	.2948	.3546	.407	.4602	.5126
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0016	.0024	.0041	.0057	.0082	.0100	.0112	.0121	.0129	.0135	.0139	.0141	.0141

TABLE 1 (continued)

$(\lambda_1,\lambda_2,\lambda_3) = (3,1,1)$ n = 23

α	.55	.6	.65	.7	.75	.8	.85	.9	.95	.975	.99	.995
t_α	.127	.256	.390	.532	.685	.858	1.060	1.319	1.714	2.069	2.5	2.807
Simu. $\hat{\alpha}$	.5594	.6130	.6646	.7158	.7646	.8194	.8644	.9106	.9556	.98	.991	.997
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0140	.0138	.0134	.0128	.0120	.0109	.0096	.0081	.0058	.004	.0027	.0015

TABLE 1 (continued)

$(\lambda_1,\lambda_2,\lambda_3) = (5,1,1)$ n = 23

α	.005	.01	.025	.05	.1	.15	.2	.25	.3	.35	.4	.45	.5
t_α	-2.807	-2.500	-2.069	-1.714	-1.319	-1.060	-.858	-.685	-.532	-.390	-.256	-.127	0.0
Simu. $\hat{\alpha}$	.0048	.0086	.0242	.0494	.1020	.1518	.2044	.2544	.3070	.3556	.4060	.4614	.5090
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0020	.0026	.0043	.0061	.0086	.0101	.0114	.0123	.013	.0135	.0139	.0141	.0141
α	.55	.6	.65	.7	.75	.8	.85	.9	.95	.975	.99	.995	
t_α	.127	.256	.390	.532	.685	.858	1.060	1.319	1.714	2.069	2.500	2.807	
Simu. $\hat{\alpha}$	.5572	.6070	.6588	.7132	.7630	.8110	.8574	.9018	.9518	.9774	.99	.9956	
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0140	.0138	.0134	.0128	.012	.0111	.0099	.0084	.0061	.0042	.0028	.0019	

TABLE 2

COMPARISON OF ASYMPTOTIC SIGNIFICANCE LEVELS
OF t WITH SIMULATED VALUES WHEN p =4

$(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = (5,1,1,1)$ $n = 23$

α	.005	.01	.025	.05	.1	.15	.2	.25	.3	.35	.4	.45	.5
t_α	-2.807	-2.500	-2.069	-1.714	-1.319	-1.060	-.858	-.685	-.532	-.390	-.256	-.127	0.0
Simu. $\hat{\alpha}$	.002	.0064	0.02	0.039	.087	.1328	.1784	.229	.2856	.3374	.3922	.4428	.4928
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0013	.0023	.0040	.0055	.0080	.0096	.0108	.0199	.0128	.0134	.0138	.0140	.0141
α	.55	.6	.65	.7	.75	.8	.85	.9	.95	.975	.99	.995	
t_α	.127	.256	.390	.532	.685	.858	1.060	1.319	1.714	2.069	2.5	2.807	
Simu. $\hat{\alpha}$	.5414	.5934	.649	.7054	.7562	.8078	.8578	.9054	.9548	.9796	.9912	.997	
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0141	.0139	.0135	.0129	.0121	.0111	.0099	.0083	.0059	.0040	.0026	.0015	

TABLE 2 (continued)

$(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = (3,1,1,1)$ $n = 23$

α	.005	.01	.025	.05	.1	.15	.2	.25	.3	.35	.4	.45	.5
t_α	-2.807	-2.500	-2.069	-1.714	-1.319	-1.060	-.858	-.685	-.532	-.390	-.256	-.127	0.0
Simu. $\hat{\alpha}$	.0016	.0038	.0136	.0346	.079	.1214	.1646	.2168	.2698	.3296	.3812	.433	.4868
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0011	.0017	.0032	.0052	.0076	.0092	.0105	.0117	.0126	.0133	.1037	.014	.0141

TABLE 2 (continued)

$(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = (3,1,1,1) \qquad n = 23$

α	.55	.6	.65	.7	.75	.8	.85	.9	.95	.975	.99	.995
t_α	.127	.256	.390	.532	.685	.858	1.060	1.319	1.714	2.069	2.5	2.807
Simu. $\hat{\alpha}$	.5392	.5968	.6520	.7102	.7568	.8092	.8644	.9138	.9612	.9814	.9944	.9978
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0141	.0139	.0135	.0128	.0121	.0111	.0097	.0079	.0055	.0038	.0021	.0013

TABLE 3

COMPARISON OF ASYMPTOTIC SIGNIFICANCE LEVELS
OF k WITH SIMULATED VALUES WHEN p= 5

$(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5) = (5,1,1,1,1)$ n = 23

α	.005	.01	.025	.05	.1	.15	.2	.25	.3	.35	.4	.45	.5
t_α	-2.807	-2.500	-2.069	-1.714	-1.319	-1.060	-.858	-.685	-.532	-.390	-.256	-.127	0.0
Simu. $\hat{\alpha}$	.0028	.0126	.0178	.0418	.0914	.1388	.1908	.2420	.2938	.3502	.4008	.4628	.5214
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0015	.0031	.0037	.0057	.0082	.0098	.0111	.0121	.0129	.0135	.0139	.0141	.0141
α	.55	.6	.65	.7	.75	.8	.85	.9	.95	.975	.99	.995	
t_α	.127	.256	.390	.532	.685	.858	1.060	1.319	1.714	2.069	2.5	2.807	
Simu. $\hat{\alpha}$	.5664	.6202	.6716	.7198	.7666	.8188	.8664	.9098	.9558	.9764	.9930	.9976	
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0140	.0137	.0133	.0127	.0120	.0109	.0096	.0081	.0058	.0043	.0024	.0014	

TABLE 3 (continued)

$(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5) = (3,1,1,1,1)$ n = 23

α	.005	.01	.025	.05	.1	.15	.2	.25	.3	.35	.4	.45	.5
t_α	-2.807	-2.500	-2.069	-1.714	-1.319	-1.060	-.858	-.685	-.532	-.390	-.256	-.127	0.0
Simu. $\hat{\alpha}$	.0012	.0032	.016	.0362	.0806	.1262	.1822	.2324	.2844	.3382	.395	.453	.513
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0010	.0016	.0035	.0053	.0077	.0094	.0109	.0119	.0128	.0134	.0138	.0141	.0141

TABLE 3 (continued)

$(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5) = (3,1,1,1,1)$ $n = 23$

α	.55	.6	.65	.7	.75	.8	.85	.9	.95	.975	.99	.995
t_α	.127	.256	.390	.532	.685	.858	1.060	1.319	1.714	2.069	2.5	2.807
Simu. $\hat{\alpha}$	.5698	.6236	.6800	.733	.782	.8314	.8802	.92	.965	.9836	.9858	.9988
$2\sqrt{\frac{\hat{\alpha}(1-\hat{\alpha})}{5000}}$	.0140	.0137	.0132	.0125	.0117	.0106	.0092	.0077	.0052	.0036	.0018	.001

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